

Problem 7: Sudoku Solver

Alex, a Sudoku enthusiast, dreams of one day participating in the prestigious World Sudoku Championship. Alex believes that mastering each type of deduction will be the key to success. So Alex practices different types each day.

Sudoku fills a 9×9 grid of cells with digits from 1 to 9. Each row of 9 must contain all digits from 1 to 9. Each column of 9 must contain all digits from 1 to 9. The grid is also divided into nine 3×3 subgrids. Each subgrid must contain all digits from 1 to 9.

A *neighbor* is any cell in the same row or same column or same subgrid. Since each digit must appear exactly once, the collective neighbors of a cell eliminate possible values for an empty cell.

An *elimination deduction* occurs when the possible values for an empty cell are reduced to one. When this happens, the cell can be filled with the unused value. For example, an empty cell could be deduced to be 1, if its neighbors include all the values from 2 to 9 (in any order). Input consists of a single 9×9 grid representing the initial state of the puzzle. The grid includes prefilled numbers (digits 1 to 9) and empty cells (represented by \emptyset).

Output the state of the grid after completing all possible elimination deductions.

Sample input 1

1	6	9	8	2	4	5	7	3
3	4	2	5	9	7	8	1	6
7	5	8	1	6	3	2	9	4
4	2	3	9	7	8	1	6	5
5	9	7	3	1	6	4	8	2
8	1	6	2	4	5	7	3	9
2	7	4	6	8	9	3	5	\emptyset
9	8	5	4	3	1	6	2	7
6	3	1	7	5	2	\emptyset	4	\emptyset

Sample output 1

1	6	9	8	2	4	5	7	3
3	4	2	5	9	7	8	1	6
7	5	8	1	6	3	2	9	4
4	2	3	9	7	8	1	6	5
5	9	7	3	1	6	4	8	2
8	1	6	2	4	5	7	3	9
2	7	4	6	8	9	3	5	1
9	8	5	4	3	1	6	2	7
6	3	1	7	5	2	9	4	8

An elimination deduction fills empty cell in row 7 with a 1. Similarly, a 9 fills the empty cell in column 7. Once both of these deductions are made the final cell is filled with 8.

Sample input 2

1	2	3	4	5	6	7	8	9
4	5	6	7	8	9	1	2	3
7	8	9	1	2	3	4	5	6
2	3	1	5	6	4	8	9	7
5	6	4	8	9	7	2	3	1
8	9	7	2	3	1	\emptyset	\emptyset	\emptyset
3	1	2	6	4	5	9	7	8
6	4	5	9	7	8	3	1	2
9	7	8	3	1	2	\emptyset	\emptyset	\emptyset

Sample output 2

1	2	3	4	5	6	7	8	9
4	5	6	7	8	9	1	2	3
7	8	9	1	2	3	4	5	6
2	3	1	5	6	4	8	9	7
5	6	4	8	9	7	2	3	1
8	9	7	2	3	1	\emptyset	\emptyset	\emptyset
3	1	2	6	4	5	9	7	8
6	4	5	9	7	8	3	1	2
9	7	8	3	1	2	\emptyset	\emptyset	\emptyset

No elimination deductions are possible; each of the empty cells has three possible values. Thus, the grid does not change.

(over)

Sample input 3

4	1	0	0	2	7	3	0	0
8	7	0	0	0	5	0	9	0
0	0	0	0	8	1	0	0	0
0	4	0	0	0	8	7	1	0
1	0	5	0	4	0	6	0	9
0	8	7	2	0	0	0	3	0
0	0	0	1	5	0	0	0	0
0	5	0	3	0	0	0	4	6
0	0	4	8	9	0	0	7	5

Sample output 3

4	1	6	9	2	7	3	5	8
8	7	2	6	3	5	4	9	1
5	9	3	4	8	1	2	6	7
3	4	9	5	6	8	7	1	2
1	2	5	7	4	3	6	8	9
6	8	7	2	1	9	5	3	4
7	6	8	1	5	4	9	2	3
9	5	1	3	7	2	8	4	6
2	3	4	8	9	6	1	7	5

After applying 46 elimination deductions, no unknown cells remain.